

MIDDLE AND HIGH SCHOOL

Truth, Beauty, and Order

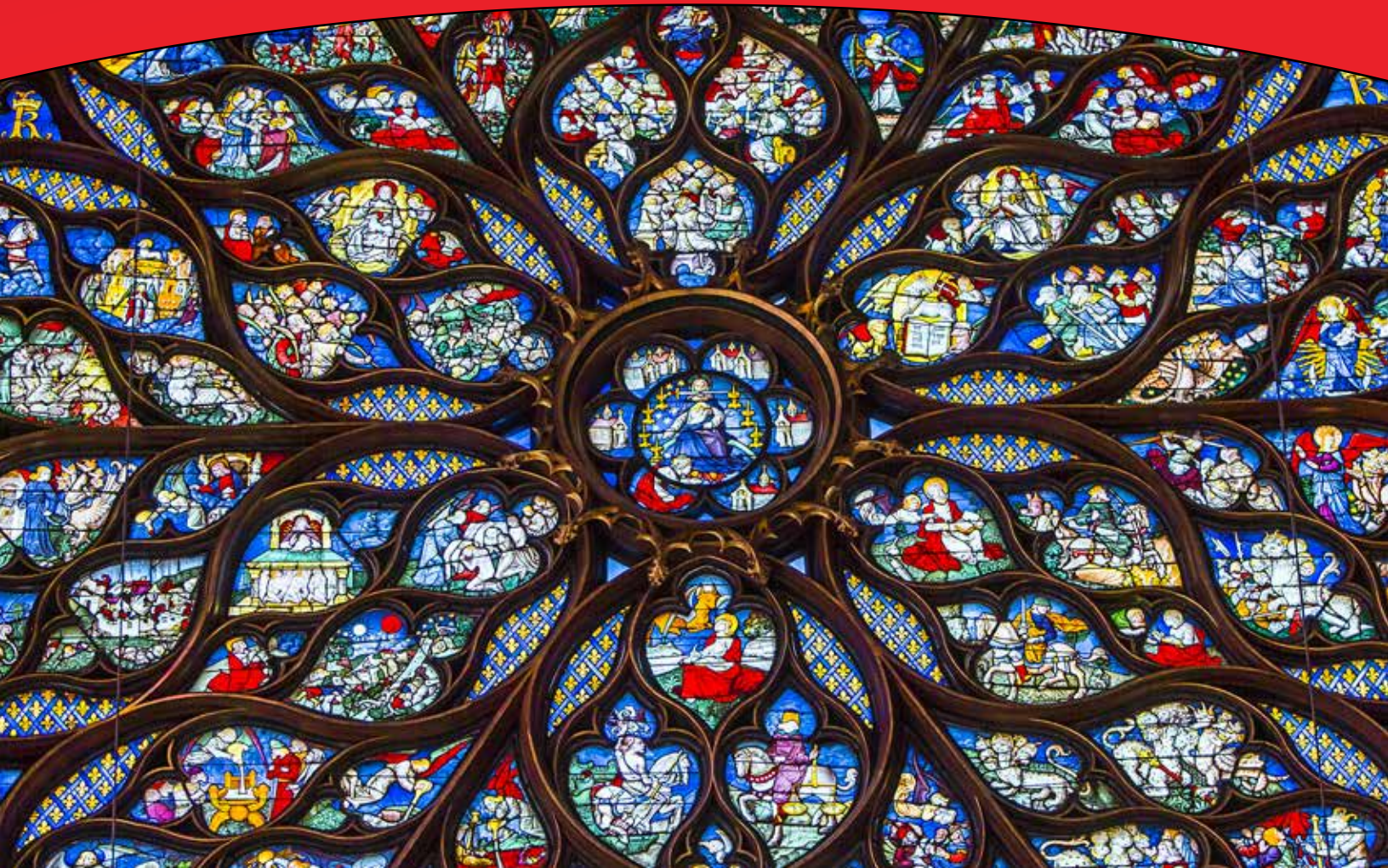
Overview

From the spirals of seashells to the soaring proportions of cathedrals, mathematics reveals the order written into creation. In this unit, students explore how patterns such as the Fibonacci sequence and the mathematical principles behind great works of art and architecture point to a universe that is intelligible, beautiful, and ordered, and to a Creator whose wisdom is reflected in the harmony of all He has made.

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Lesson 1

The Fibonacci Sequence: Mathematical Order in Nature



Lesson 1 at a Glance



Learning Goals

- The universe is ordered because it comes from God's Logos (reason).
- Human beings are capable of discovering the order of the universe because we are made in the image of God.
- Mathematical order is not an illusion or social construct imposed upon a meaningless reality.
- The beauty and harmony of creation reflect the infinite beauty of the Creator and ought to inspire in us awe and wonder.
- The golden spiral is a special spiral that gets wider at a constant rate based on the golden ratio, a number that has fascinated mathematicians, artists, and architects for centuries. It is often associated with beauty and harmony and can be seen in some patterns in nature, art, and design.
- Because truth and beauty come from God, studying mathematics can become a way to contemplate the divine.



Cardinal Newman Society Mathematics Standards

- **M.712 DS1** Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.
- **M.712 DS2** Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics
- **M.712; GS4:** Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.
- **M.712; DS6:** Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.

Lesson 1 at a Glance, continued



Lesson Plan Chart

Homework Reading: Math, Wonder, and Wisdom

Warm-Up: Teacher-guided activity: Any Two Numbers Converge on Phi

- You will need: Graph paper, image at [SophiaOnline.org/FibonacciSequence](https://www.sophiaonline.org/FibonacciSequence)
Optional: Fibonacci items (see preparation before class)

Activity: Fibonacci Sequence and Vi Hart Video: Doodling in Math: Spirals, Fibonacci, and Being a Plant [Part 1 of 3]

- You will need: PowerPoint available at [SophiaOnline.org/TruthBeautyOrder](https://www.sophiaonline.org/TruthBeautyOrder); video at [SophiaOnline.org/ViHart](https://www.sophiaonline.org/ViHart)

Formative Assessment: Wisdom and Wonder



Connections to the Catechism in This Lesson

CCC 36, 283, 341, 353



Vocabulary Your Students Will Learn

- The **Fibonacci Sequence** is a sequence where every number after the first two is the sum of the two numbers before it. When Fibonacci numbers get larger, the ratio between neighboring numbers approaches the Golden Ratio (about 1.618)
- The **Golden Ratio** is a mathematical pattern where two parts fit together in a way that creates a sense of balance and harmony. Its value is about 1.618, and it has fascinated mathematicians, artists, and architects for centuries
- **Wisdom** is the virtue that allows us to see the world as God does. It is knowing what is true and good, judging rightly, and choosing to live according to God's plan.

Lesson 1 at a Glance, *continued*



Necessary Math Background Knowledge

- **Division** is the mathematical operation of separating a quantity into equal groups or finding how many times one number fits into another.
- A **ratio** is a way of comparing two quantities by showing how much of one thing there is compared to another.



Key Scripture Passages

For from the greatness and the beauty of created things
their original author, by analogy, is seen.

— *Wisdom 13:5*

For what can be known about God is evident....because God made it evident.... Ever since the creation of the world, his invisible attributes of eternal power and divinity have been able to be understood and perceived in what he has made.

— *Romans 1:19-20*



Prayer for This Lesson

Creator of all things, true source of light and wisdom, origin of all being, graciously let a ray of your brilliance penetrate into the darkness of our understanding

— *St. Thomas Aquinas*

Day 1

Background/Homework

Have students read the Background Essay: Wonder and Wisdom, discuss the questions and do the simple measurements with their parents.



Preparation Before Class

If possible, gather and bring some pinecones, sunflowers, whole pineapples, daisies, succulent plants, and/or other objects which demonstrate the Fibonacci sequence.

Warm-Up

- A.** Begin with the prayer for the lesson, which is from St. Thomas Aquinas's "Prayer Before Study," or another prayer of your choosing.
- B.** Write on the board the quote from Romans 1:19–20:
For what can be known about God is evident....because God made it evident.... Ever since the creation of the world, his invisible attributes of eternal power and divinity have been able to be understood and perceived in what he has made.
- C.** Ask the class what they think St. Paul meant about God in this passage. *Allow for reasoned discussion. Students may relate it to how we can learn about God through things we can see and experience in this world: the beauty of the world, the grandeur of nature, the depths of the ocean, the vastness of space, the love of a family, and so forth. Ask if they have ever thought God might be showing us a sign of Himself in math. Pause and ask them to keep this possibility in mind as you proceed through the lesson.*
- D.** Then, ask for a volunteer to call out any two numbers — they can be completely random. Take the first two numbers a student offers. Write the first two numbers on the board. Then add them together to get a third number. Keep adding the last two numbers to get the next one. Do this about 10–12 times, going slowly, inviting different students to call out the next number each time.
- E.** After you have about 12 numbers on the board, ask students to divide the last number by the one just above it. The result should be near 1.618. Do it one more time with the two numbers above that, and you will get the same or a very similar answer.
- F.** Explain that no matter what two numbers we started with (recall the original pair of numbers a student picked randomly off the top of their head), if we just keep adding, the ratio between consecutive numbers homes in on the same target every single

time: 1.618. This number is so significant that it has several names, including “the Golden Ratio” or *Phi*, represented by the Greek letter ϕ .

- G.** You may wish to further demonstrate the point by picking two dramatically different numbers (for example, 1 and 1,000,000) and running through the exercise again quickly.
- H.** Explain that mathematicians call *phi* an “attractor”. It draws sequences toward itself, no matter where they start. Why does this number exist at all? Why should there be a ratio that nature returns to again and again, that shows up in flowers and DNA and galaxies? Did someone put it there?
- I.** Continue to explain that the series of numbers we have been working with is called the Fibonacci sequence (1, 1, 2, 3, 5, 8, 13...). We have shown how dividing consecutive numbers in this sequence converges on $\phi \approx 1.618$, the golden ratio.
- J.** Invite the class to render the Fibonacci Sequence on graph paper. Begin with 1 and 1 - draw two squares on the grid paper, adjacent to each other. Next to them, draw a 2x2 square, and, next to that, a 3x3 square (i.e., the sum of the previous two). Continue with this pattern with the Fibonacci numbers and see how many squares they can draw.
- K.** When students have filled their paper, have them draw a curved line through the opposite corners of each square. Ask them if they can see a spiral pattern in what they drew.
- L.** To help students visualize this, show them the image found at SophiaOnline.org/FibonacciSequence.

Activity

- A.** Using the PowerPoint available at SophiaOnline.org/TruthBeautyOrder, show the images on slides 2–4.
- B.** Then show the class the eight-minute video titled “**Doodling in Math: Spirals, Fibonacci, and Being a Plant [Part 1 of 3]**” available at Sophiaonline.org/ViHart.
- C.** If possible, display the objects you brought in and allow your students to examine them.
- D.** Then summarize the following key points:
 - **The Golden Spiral:** It is a special spiral that gets wider at a constant rate based on the golden ratio. It is often associated with beauty and harmony and can be seen in nature from nautilus shells to hurricanes to spiral galaxies.
 - **Fibonacci Numbers in Plant Leaf Arrangement:** The number of leaves, petals, or branches that appear at each level of a plant as you spiral up the stem is almost always a Fibonacci number. It is the mathematically optimal way to arrange leaves so each one gets maximum sunlight without being blocked by the one above it. Nature keeps arriving at the same solution!

- **Flower Petals:** Count the petals on a daisy, a buttercup, a lily, or a delphinium. You almost always get Fibonacci numbers: 3, 5, 8, 13, 21, 34. It is not every flower, but it is consistent enough to be striking.
 - **The Fibonacci Sequence itself and the Golden Ratio:** As we did earlier, you can take any two numbers, add them together, keep going the same way, and the ratio of consecutive numbers will always converge on 1.618. It does not matter what numbers you start with.
 - **DNA Molecule Dimensions:** A DNA double helix measures 34 angstroms in length per full twist and 21 angstroms wide. Both are Fibonacci numbers, and $34/21 = 1.619$ is almost exactly the golden ratio.
 - **The Golden Rectangle's Self-Similarity:** If you take a golden rectangle (one with sides 1 and 1.618) and cut a square off one end, the remaining piece is another perfect golden rectangle. You can do this forever: it is a geometric truth.
- E.** Close by emphasizing that the golden spiral is a mathematical ideal. Nature often produces spirals that are similar because many living things grow by adding new material while keeping the same overall shape. Some of these patterns are closely connected to the Fibonacci sequence and the golden ratio, while others simply resemble the golden spiral without matching it exactly.
- E.** Ask the class if they found the golden ratio or a ratio close to it on their homework last night. Explain that the point of all we have been learning is not that everything contains the golden ratio, but rather that mathematics reveals order and harmony in creation.

Formative Assessment

Invite students to journal a response on **Wisdom 13 Reflection**.

The Fibonacci Sequence: Mathematical Order in Nature

LESSON OVERVIEW



Vocabulary

- The **Fibonacci Sequence** is a sequence where every number after the first two is the sum of the two numbers before it. When Fibonacci numbers get larger, the ratio between neighboring numbers approaches the Golden Ratio (about 1.618)
- The **Golden Ratio** is a mathematical pattern where two parts fit together in a way that creates a sense of balance and harmony. Its value is about 1.618, and it has fascinated mathematicians, artists, and architects for centuries
- **Wisdom** is the virtue that allows us to see the world as God does. It is knowing what is true and good, judging rightly, and choosing to live according to God's plan.



Prayer for This Lesson

Creator of all things, true source of light and wisdom, origin of all being, graciously let a ray of your brilliance penetrate into the darkness of our understanding

—St. Thomas Aquinas

Math, Wonder, and Wisdom

You have taken many math classes in your life, but have you ever wondered: what does math *actually* do? What actual significance is there in doing math problems, in adding two quantities, or solving for x ? Do numbers and the relationships between them really mean anything for us and the world around us? Why are there numbers or shapes at all? Why do sunflowers grow in beautiful spirals, why do snowflakes form such intricate designs, why do the stars and planets move in orderly paths across the sky?



The answer is that the universe is not a meaningless accident. It was made by God, who is Wisdom itself. The world is full of order because it was created through God's eternal Word — the Logos — who gives meaning and harmony to all things. Human beings are capable of discovering that order because we are made in the image of that same *Logos*!

Human beings are set apart from all other material creation in that we have intellects, or minds, that are like God's. God made you in His image. Your mind has been literally designed to think and reason, to recognize patterns, to seek truth and goodness, and to delight in beauty. That is why mathematics is not merely a system of symbols or rules that people made up. Math is about discovery, not invention. When we discover a mathematical principle, we are uncovering something real that has been woven into creation from the very beginning. We are glimpsing the mind of God!

The world is filled with signs of God's wisdom, waiting for us to notice them. Math is in everything around us. Every movement of the wind, every wave in the ocean, every pattern in every leaf, every beautiful symmetry in creation is a clue pointing beyond itself to the Creator.

The more we learn about the order of creation, the more we discover that the universe is not cold, random, or meaningless, but full of purpose, proportion, and beauty. And when we see that beauty — whether in a spiral shell,

a great redwood tree, or the stars above us — it should awaken in us a sense of awe and wonder. Wonder is often the beginning of wisdom, and every discovery

in this world is a glimpse of our ultimate destiny: contemplating the One who made all things.

- 1
- What are some beautiful things in or around your home? Can you see anything beautiful outside your windows?
- 2
- Do you think humans tend to design objects with proportions that feel balanced and beautiful? Why or why not?

Using a ruler, record this information for each of the following objects:

Object	Length	Width	Length/Width
Beautiful thing inside our home			
Beautiful thing outside near our home			
Credit card			
Index card			
Picture frame			
Book			
Smartphone			

Wisdom 13 Reflection

Directions: Read the Scripture passage and answer the reflection questions.

Wisdom 13:1–5

Foolish by nature were all who were in ignorance of God,
and who from the good things seen did not succeed in
knowing the one who is,
and from studying the works did not discern the artisan;

Instead either fire, or wind, or the swift air,
or the circuit of the stars, or the mighty water,
or the luminaries of heaven, the governors of the world, they
considered gods.

Now if out of joy in their beauty they thought them gods,
let them know how far more excellent is the Lord than these;
for the original source of beauty fashioned them.

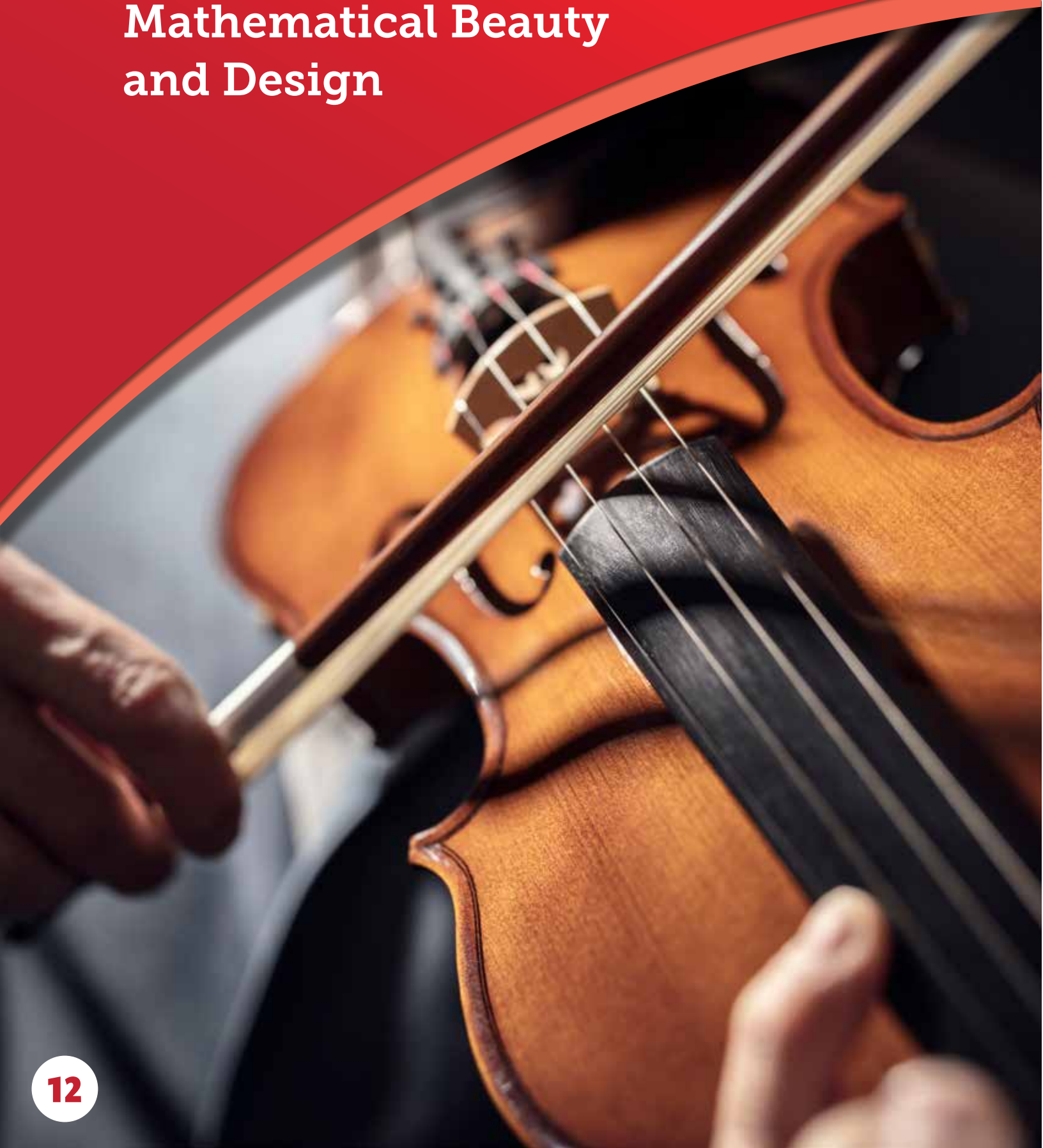
Or if they were struck by their might and energy,
let them realize from these things how much more powerful
is the one who made them.

For from the greatness and the beauty of created things
their original author, by analogy, is seen...

- 1 In what ways does the sacred author distinguish between worshipping the Creator, and worshipping His creation?
- 2 Who does the sacred author call the original source of beauty?
- 3 Think back over our lesson today on the golden ratio and its prevalence in nature. Did it surprise you to discover how nature is so carefully ordered? Why or why not? What does this unique pattern that pervades nature reveal to us about God?

Lesson 2

Art, Architecture, and Music: Mathematical Beauty and Design



Lesson 2 at a Glance



Introduction

In Lesson 1, students discovered that mathematical order — the Fibonacci sequence and the golden ratio — appears throughout the natural world. Lesson 2 turns from what God has created to what human beings make. Students explore how architects, artists, and musicians have used ratio and proportion, sometimes deliberately and sometimes intuitively, to make beautiful works.



Learning Goals

- Human beings, made in the image of God, not only discover mathematical order in the world around them but also express that order in things we make.
- Throughout history, architects, artists, and musicians have used ratio and proportion to create beautiful works.
- St. Thomas Aquinas identified three elements of beauty: integrity or wholeness (*integritas*); proportion or harmony (*consonantia*); and clarity or radiance (*claritas*).
- Human creativity, oriented towards truth, beauty, and goodness, is a genuine participation in the wisdom and craftsmanship of the Creator.
- Because truth and beauty come from God, studying the mathematics behind art, architecture, and music can become a way to contemplate the divine



Cardinal Newman Society Mathematics Standards

- **M.712 DS1** Display a sense of wonder about mathematical relationships, especially mathematical certitude which is independent of human opinion.
- **M.712 DS2** Share with others the beauty, harmony, proportion, radiance, and wholeness present in mathematics.
- **M.712; GS4:** Explain how mathematics in its reflection of the good, true, and beautiful reveals qualities of being and the presence of God.
- **M.712; DS6:** Propose how mathematical objects or proofs (such as the golden mean, the Fibonacci numbers, the musical scale, and geometric proofs) suggest divine origin.

Lesson 2 at a Glance, continued



Lesson Plan Chart

Homework Reading: Ratio, Reason, and Beauty

Warm-Up: Teacher-guided activity: The Sound of a Ratio

You will need:

- a stringed instrument (guitar, ukulele, violin) or a simple one-string monochord (a ruler, a rubber band, and a small box); alternatively, a free online string simulator app
- Videos at SophiaOnline.org/SisterAct1, SophiaOnline.org/AncientMathandMusic and SophiaOnline.org/MusicMath.

Activity: Teacher-guided activity: Ratio and Proportion in Beautiful Art, Architecture, and Music

- PowerPoint available at SophiaOnline.org/DenverMath

Formative Assessment: Measure, Number, and Weight Reflection

- You will need: Video at SophiaOnline.org/SisterAct1



Connections to the Catechism in This Lesson

CCC 299, 341, 1156, 2501–2502



Vocabulary Your Students Will Learn

- **Proportion** is the relationship between the size of one part and another part (or the whole), often expressed as a ratio. Proportion (*consonantia*) is one of three elements of beauty identified by St. Thomas Aquinas.
- **Consonance** is a combination of musical tones that sounds pleasing, stable, or “in agreement” to the ear. It is typically produced by tones whose frequencies form a simple whole-number ratio.
- An **interval** is the distance in pitch between two musical notes. We can describe intervals using the ratio between the two notes’ frequencies.

Lesson 2 at a Glance, *continued*



Necessary Math Background Knowledge

- A **ratio** is a way of comparing two quantities by showing how much of one thing there is compared to another.
- **Simplifying** a ratio means reducing it to its lowest terms (for example, 8:4 simplifies to 2:1), which makes it easier to compare one ratio to another.



Key Scripture Passages

You have disposed all things by measure and number and weight.

— *WISDOM 11:20*

I have filled him with a divine spirit of skill and understanding and knowledge in every craft ...

— *EXODUS 31:3*



Prayer for This Lesson

(Lead your students in the following prayer.)

Creator of all things, true source of light and wisdom, origin of all being, graciously let a ray of your brilliance penetrate into the darkness of our understanding

— *St. Thomas Aquinas*

Day 1



Preparation Before Class

If possible, bring a stringed instrument (guitar, ukulele, or violin), and/or materials to build a simple one-string monochord (a ruler, a rubber band, and a small box or hardcover book). If you have a student who plays any of these instruments, you might invite him or her to bring it to class and help lead the warm-up activity. If none of these are possible, you can use a free string instrument simulator app.

Warm-Up

- A.** Begin with the prayer for the lesson, from St. Thomas Aquinas's "Prayer Before Study," or another prayer of your choosing.
- B.** Write on the board the quote from Wisdom 11:20: "... you have disposed all things by measure and number and weight."
- C.** Ask the class: last lesson, we found the same ratio — the golden ratio — turning up again and again in nature. Do you think human beings, when we build or make something beautiful, also tend to use number and measure (whether or not it's on purpose)? Are there things we all are meant to recognize right away as beautiful, or not beautiful?
Allow for brief discussion.
- D.** Play the brief video (showing a rehearsal scene from the movie *Sister Act*) available at **SophiaOnline.org/SisterAct1**. Ask the class to discuss briefly why the choir sounds bad. *They may say they sound "out of key" or "off," etc.*
- E.** After allowing discussion for about a minute, let the class know you're going to play a pair of sounds and you'd like them, for each pair, to raise their hand for the one sounds more beautiful.
- F.** Ask students to close their eyes. Then, using your stringed instrument, play a perfect fifth (instructions below). Pause for a few seconds, and then pluck the open string, followed by a random spot that isn't a clean fraction, e.g. roughly $\frac{8}{15}$ of the way down. Allow a few seconds for students to mentally compare the two sets of tones. Then play them each again, asking the class to raise their hand for the tone they think sounds better. Repeat this process a few times and count the votes each time.
 - To play a perfect fifth on the stringed instrument: Pluck the open string; then hold the string at two-thirds of its length and pluck it.
- G.** Let the class know they can open their eyes and then discuss briefly the votes. More students probably thought the fifth sounded better.

- H.** With their eyes now open, show how intervals works with your stringed instrument (either by yourself or your student musician volunteer.) Start with an octave. Pluck the full open string and let students listen. Then, holding the string at exactly half its length, pluck it again. Ask students to describe what they hear. *They might say it sounds like “the same note,” only higher.*
- I.** Then ask if anyone can explain what math has to do with what we just did. You might offer a hint: think about the length of the string that was vibrating. *An octave is a ratio of 2:1. When you held the string at the halfway point, the part of the string that was vibrating at twice the frequency as when you plucked the open string.*
- J.** Allow discussion, and then explain that we have been experiencing ratios, or proportion. Proportion is the relationship between the size of one part and another part (or the whole), often expressed as a ratio. (i.e. the pairs of notes we heard came from plucking pairs of string lengths of different proportions.)
- K.** Recall that the ratio of the two string lengths you just plucked is 2:1; this is called an octave. Check understanding by asking, if an open string is a C note, and I pluck it again at exactly half it's length, what note will I get? *The answer is C – i.e. the same note, but higher: a higher C.* Now hold the string at 2/3rds of its length and pluck it. Ask students to compare this sound to the first two. Be sure to point out the relative length of the two segments of the string. Explain that this interval is called a fifth, and its ratio is 3:2. Now pluck it three fourths of its length. This is called a fourth, and its ratio is 4:3.
- L.** Now, offer the counter example again. After plucking the string at the half, two-thirds, and three-quarters marks (octave, fifth, fourth), pluck it again somewhere that isn't a clean fraction.
- M.** Ask students to compare what they hear to the earlier three (octave, fifth, fourth). It won't “agree” the way the others did, and that's the point. Explain that they may lack words to describe why some combinations (i.e. ratios) sound “better” or “worse.” And that is okay. Our activity is getting at a longing, or a natural attraction to beauty (including beautiful proportion in sound that St. Thomas called consonance) that God has given us! Consonance is a combination of musical tones that sounds pleasing, stable, or “in agreement” to the ear — typically produced by tones whose frequencies form a simple whole-number ratio. St. Thomas Aquinas taught that proportion (along with integrity and clarity), was essential to beauty. In other words, beauty was not just personal preference or “in the eye of the beholder,” but something real and objective.
- N.** Continue to explain that about 2,500 years ago, the Greek mathematician Pythagoras discovered this same pattern: the musical intervals people find most pleasing correspond to simple whole-number ratios. This discovery became a foundation of music theory for the next 2000 years.

- O.** Play the videos available at **SophiaOnline.org/AncientMathandMusic** and **SophiaOnline.org/MusicMath** and allow for brief discussion after each.
- P.** As is practical, let students examine the instrument and try the string-length ratios themselves.

Activity

- A.** Using the PowerPoint available at **SophiaOnline.org/DenverMath**, show the images and summarize the following key points:
 - **Whole-Number Ratios in Classical Architecture:** Ancient Greek and Roman builders, following principles later written down by the architect Vitruvius, used simple whole-number ratios — such as 1:2, 2:3, and 4:9 — to proportion buildings. Scholarly reconstructions of the Parthenon, for example, find that its footprint approximates a 9:4 ratio, a relationship that recurs at several scales throughout the design.
 - **Proportion in Gothic Cathedrals:** Medieval masons used geometric methods called *ad quadratum* (nesting rotated squares) and *ad triangulum* (nesting triangles) to lay out floor plans, elevations, and window tracery, generating consistent ratios throughout a building. This is documented not just by the finished cathedrals, but by surviving medieval drawings, prick-marks left by masons' compasses, and instructional handbooks.
 - **Leonardo da Vinci and the “Divine Proportion”:** Leonardo illustrated the geometric solids for the mathematician Luca Pacioli's 1509 book *De Divina Proportione*. This collaboration is well documented and reflects a real and rich Renaissance interest in geometry and proportion — even where specific popular claims about individual paintings go beyond the evidence historians have found.
 - **Pythagoras and the Music of Ratios:** As we have learned, Pythagoras discovered that the musical intervals people across cultures find most pleasing — the octave (2:1), the perfect fifth (3:2), and the perfect fourth (4:3) — correspond to simple whole-number ratios of string length or frequency. This discovery shaped Western music theory for over two thousand years.
- B.** Close by emphasizing that architecture and music are made by human choice, not grown the way a plant or shell grows, yet builders, artists, and musicians keep arriving at many of the same ratios nature does. Ask the class why they think that might be. *Connect this to the Catechism's teaching that human beings, made in the image of God, have a share in the Creator's own wisdom and creativity when they make something well and beautifully (CCC 2501–2502).*
- C.** Ask the class to share the ratio they found on their homework measuring table, and whether any came close to a “special” ratio. Reiterate the same caution from Lesson 1: the point is not that every beautiful piece of music must contain a “magic interval,” or secret chord, but that human beings, made in the image of God, are drawn again and again toward a set of proportions that convey order and harmony.

Formative Assessment

- A.** Invite students to journal a response on **Measure, Number, and Weight Reflection**. Create the Venn Diagram on the board, if possible, and fill it out together. In addition to the ideas students generate, you might add:
- **Architecture:** *Understood through sight, touch*
 - **Music:** *Understood through hearing*
 - **Both:** *Evoke emotions, communicate ideas, when done well, include beautiful proportions. (and possibly understood via hearing as well, e.g. acoustics of a space)*
- B.** Close by playing a short video of the same choir now singing in perfect harmony, available at **SophiaOnline.org/SisterAct2**. Be sure to pause the video at the end of the *Salve Regina*, at 1:09.

Art, Architecture, and Music: Mathematical Beauty and Design

LESSON OVERVIEW



Vocabulary

- **Proportion** is the relationship between the size of one part and another part (or the whole), often expressed as a ratio. Proportion (consonantia) is one of three elements of beauty identified by St. Thomas Aquinas.
- **Consonance** is a combination of musical tones that sounds pleasing, stable, or “in agreement” to the ear. It is typically produced by tones whose frequencies form a simple whole-number ratio.
- An **interval** is the distance in pitch between two musical notes. Musicians often describe intervals using the ratio between the two notes’ frequencies.



Prayer for This Lesson

Creator of all things, true source of light and wisdom, origin of all being, graciously let a ray of your brilliance penetrate into the darkness of our understanding

—St. Thomas Aquinas

Ratio, Reason, and Beauty

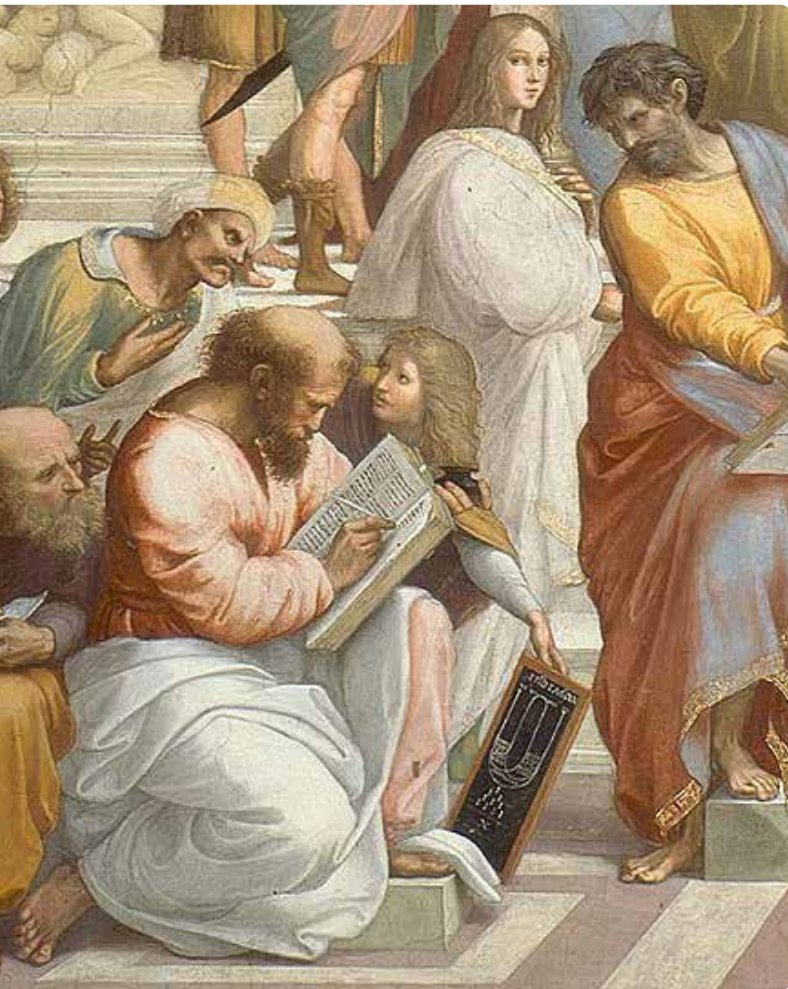
Directions: Read the Essay and discuss the conversation questions with your family.

You have probably walked into a room, or looked at a painting, or heard a piece of music, and simply felt: this is beautiful. But have you ever wondered why? What is it about a certain building, melody, or painting that makes it satisfying to look at or listen to, while another arrangement of the very same materials (whether stone, pigments, or sound) leaves us cold?

One answer is that, across very different cultures and centuries, human beings have been drawn to a set of proportions: the relationship between the parts of a thing and the whole. In the previous lesson, you discovered that nature itself is full of mathematical order — spirals, sequences, and ratios that no one invented but that everyone can discover. Now we are looking at this phenomenon from a different angle: when human beings make something, from a cathedral, to a sculpture, to a song, do we use number and measure in a similar way? And if so, where does that impulse come from?

The answer, historically, is yes. About 2,500 years ago, the Greek philosopher and mathematician Pythagoras discovered that plucking a string at different lengths produces sounds that sound pleasing or not — i.e. which either clash or agree. He learned that the sounds that agree — what musicians call consonance — correspond to simple whole-number ratios: 2:1, 3:2, 4:3. This was not a matter of personal taste or culture; it was a mathematical fact about sound waves that Pythagoras uncovered, and it has shaped music theory ever since.

In a similar way, medieval master masons used compass-and-straightedge geometry — nesting squares and triangles — to plan the floor and elevation of Gothic cathedrals, generating harmonious



School of Athens (detail), by Raphael

proportions throughout a single building. Renaissance artists and mathematicians, including Leonardo da Vinci, took a serious, sustained interest in geometry and proportion as tools for beauty, collaborating on treatises like Luca Pacioli's *De Divina Proportione*.

So where does our ability to make something beautiful come from? Scripture offers an answer. In the book of Exodus, when God calls for the building of a place of worship in the desert, He does not do it himself, or even hand down a finished design. God calls a craftsman named Bezalel and fills him "with a divine spirit of skill and understanding and knowledge in every craft" (Exodus 31:3). Thanks to God's gift, Bezalel himself can design and build with his own hands and his own mind. Human skill is one of the ways God plans for His own wisdom to reach the world, that is, through minds and hands made in His image. The *Catechism* explains that because we are made in the image of God, our own artistic work "bears a certain likeness to God's activity in what he has created" (CCC 2501).

The mathematics behind art, architecture, and music demonstrates this truth. Whether we are counting flower petals, listening to a beloved song



The Goldsmith, by James Tissot

like "Somewhere over the Rainbow," or measuring a cathedral door, we keep encountering the same small set of ratios that human beings, across history, have found beautiful. We are participating, in our own small and creaturely way, in an order and a beauty that was there before us, and that comes from God.

Conversation Questions

- 1 Think of something beautiful that someone you know has made — a drawing, a song, a garden, a meal, a piece of furniture, a building. What skill, care, or thought do you think went into it?
- 2 What makes a thing that someone has created beautiful?
- 3 Do you think the ability to create beautiful things is a gift, a skill a person learns, both, or something else? Explain your answer.

Using a ruler record the following with your family. (You can skip this activity if you completed Lesson One)

Object	Length	Width	Length/Width
A door in your home	_____	_____	_____
A window	_____	_____	_____
A book you own	_____	_____	_____
A picture frame	_____	_____	_____
A smartphone or tablet	_____	_____	_____
A piece of furniture (table, dresser, etc.)	_____	_____	_____

Listening

Use a stringed instrument, (e.g. a guitar, or a rubber band stretched over a box or ruler) or a stringed instrument simulator app to do the following activity:

- First, pluck the full length of the string and listen.
- Then hold the string at the halfway point, and pluck it again.
- Next, hold it at the two-thirds point, and pluck it again.
- In a sentence or two, describe what you hear each time.

Measure, Number, and Weight Reflection

Directions: Read the Scripture passage and answer the reflection questions.

Wisdom 11:20–22

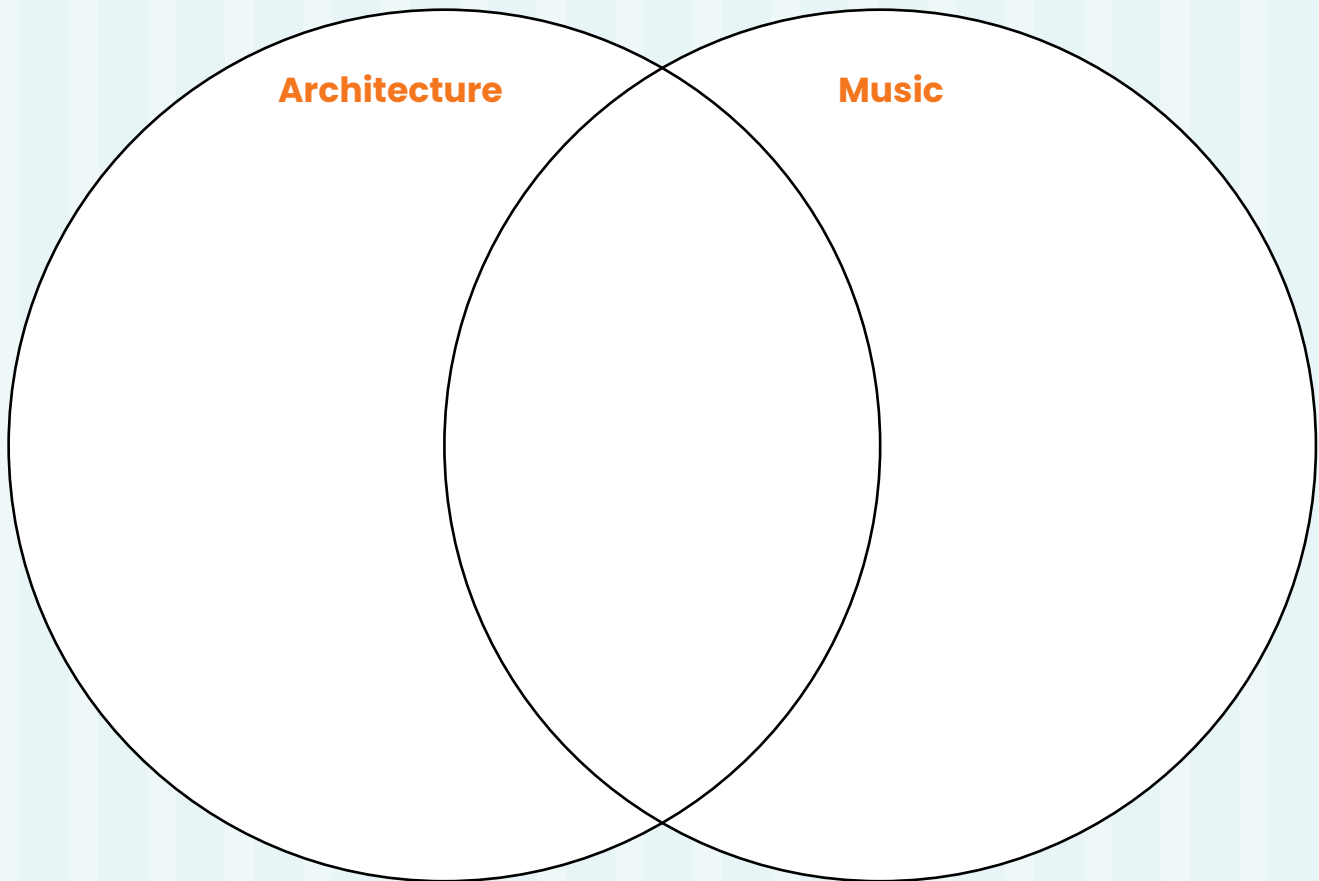
Even without these, they could have been killed at a single blast, pursued by justice and winnowed by your mighty spirit. But you have disposed all things by measure and number and weight.

For great strength is always present with you; who can resist the might of your arm? Indeed, before you the whole universe is like a grain from a balance, or a drop of morning dew come down upon the earth.

Exodus 31:1–5

The LORD said to Moses: See, I have singled out Bezalel, son of Uri, son of Hur, of the tribe of Judah, and I have filled him with a divine spirit of skill and understanding and knowledge in every craft: in the production of embroidery, in making things of gold, silver, or bronze, in cutting and mounting precious stones, in carving wood, and in every other craft.

- 1 What does it mean to say that God “disposed all things by measure and number and weight”?
- 2 The German writer Goethe referred to architecture as “frozen music.” Fill in a simple Venn diagram comparing architecture and music and/or write a brief journal response to this idea.



- 3 In Exodus 31, God fills Bezalel “with a divine spirit of skill and understanding and knowledge in every craft” to help build a place of worship. What does this suggest about where human artistic and creative ability comes from? What does it suggest about how God views us, His creation?
- 4 Think back over today’s lesson on ratio and proportion in art, architecture, and music. Did it surprise you that so many different human creations rely on the same handful of mathematical relationships? What might that reveal about the people who made them — and about God?